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Experimental study of multiple-shot unitary channels discrimination using the IBM Q computers

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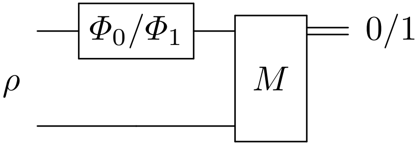
- Jan Hlisnikovský
- Tomáš Bezděk
- Paulina Lewandowska
- Ryszard Kukulski

What is Quantum Channel Discrimination?



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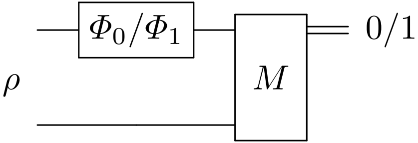

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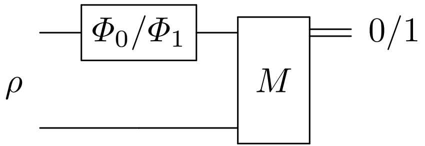

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- Earliest theoretical work: **single shot state discrimination**, solved by Helstrom in 1969.
- Recent practical experiments: **Distinguishing unitary gates on IBM Q** by S. Liu.

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- **Our main motivation is to test the performance of various theoretically explored schemes on real hardware such as IBM Q devices.**

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  - $\Phi \otimes \mathbb{1}_{\mathsf{L}(\mathcal{Z})} : \mathsf{PSD}(\mathcal{X} \otimes \mathcal{Z}) \to \mathsf{PSD}(\mathcal{Y} \otimes \mathcal{Z})$ are called completely positive (CP).
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- Such CPTP maps are called **quantum channels** and we denote them $C(\mathcal{X}, \mathcal{Y})$.
- A **Positive Operator-Valued Measure** (POVM) $\mathcal{P}$ is a collection of operators $\{E_0, \cdots, E_n\} \subset \mathsf{PSD}(\mathcal{X})$ with property $\sum_{i=0}^{n} E_i = \mathbb{1}$.
  The probability of obtaining E_i after measuring ρ is given by Born rule $\mathsf{Tr}(E_i \rho)$.

Numerical range, arc function and diamond norm

- For any operator $X \in \mathsf{L}(\mathcal{X})$, the **numerical range** is defined as:

$$\mathsf{W}(X) := \{\langle x| X |x\rangle : |x\rangle \in \mathcal{X}, \langle x|x\rangle = 1\}.$$

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$$\theta(U) := \min\left\{\Delta \in [0, 2\pi) : \exists\, \alpha \text{ s.t. } \mathsf{spec}(U) \subset \{e^{i\theta} : \theta \in [\alpha, \alpha + \Delta]\}\right\}.$$

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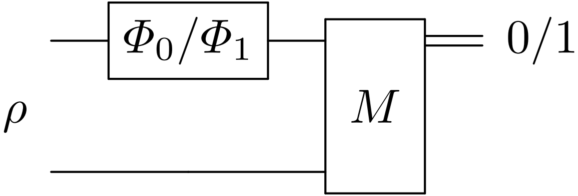
- The **diamond norm** $\|\cdot\|_\diamond$ for $\Phi : \mathsf{L}(\mathcal{X}) \to \mathsf{L}(\mathcal{Y})$ is:

$$\|\Phi\|_\diamond = \left\|\Phi \otimes \mathbb{1}_{\mathsf{L}(\mathcal{X})}\right\|_1,$$

  where $\|\Phi\|_1 = \max\{\|\Phi(X)\|_1 : X \in \mathsf{L}(\mathcal{X}), \mathrm{Tr}(X^\dagger X) \le 1\}$

Probability of successful channel discrimination

- We are given black-box quantum channel Φ which is either Φ_0 or Φ_1 $(\in \mathrm{C}(\mathcal{X}, \mathcal{Y}))$.
- To determine which was applied, we prepare an input state $\rho \in \Omega(\mathcal{X} \otimes \mathcal{Z})$ and perform a binary measurement $\{E_0, E_1\} \subset \mathsf{PSD}(\mathcal{Y} \otimes \mathcal{Z})$ after applying $\Phi \otimes \mathbb{1}_{\mathsf{L}(\mathcal{Z})}$.



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- The success probability derived from Born's rule:

$$\begin{aligned} p_{\mathsf{succ}} = & \frac{1}{2} \mathsf{Tr}\big(E_0 (\Phi_0 \otimes \mathbb{1}_{\mathsf{L}(\mathcal{Z})})(\rho)\big) \\ + & \frac{1}{2} \mathsf{Tr}\big(E_1 (\Phi_1 \otimes \mathbb{1}_{\mathsf{L}(\mathcal{Z})})(\rho)\big). \end{aligned}$$

- By the Holevo-Helstrom theorem, p_{succ} can be expressed using the diamond norm:

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- **The goal is to choose ρ and $\{E_0, E_1\}$ to maximize p_{succ}.**

Single-shot discrimination of unitary channels

- **Perfect discrimination** is situation whenever $p_{\mathsf{succ}} = 1$.
- **Unitary channels** are maps $\Phi_U \in \mathrm{C}(\mathcal{X})$ for which $\Phi_U(X) = UXU^\dagger$ where $U \in \mathsf{L}(\mathcal{X})$ is the unitary matrix

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- **Perfect discrimination** $0 \in W(V^\dagger U) \Leftrightarrow \exists \ket{\psi}$ such that $\bra{\psi} V^\dagger U \ket{\psi} = 0$.
- At the same time $0 \in W(V^\dagger U) \iff \theta(V^\dagger U) \geq \pi$
- **Equivalence:** $\theta(V^\dagger U) \geq \pi \Leftrightarrow 0 \in W(V^\dagger U) \Leftrightarrow \nu = 0 \Leftrightarrow \|\Phi_U - \Phi_V\|_\diamond = 2 \Leftrightarrow p_{\text{succ}} = 1$

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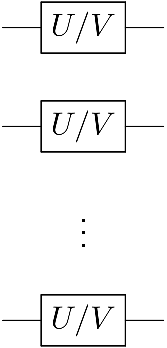

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- The **parallel strategy**: apply $U^{\otimes N}$ and compare with $V^{\otimes N}$.
- Arc function scales:

$$\theta((V^{\otimes N})^\dagger U^{\otimes N}) = N\theta(V^\dagger U), \text{ if } N\theta(V^\dagger U) < 2\pi$$

- Perfect discrimination $\Leftrightarrow N\theta(V^\dagger U) \geq \pi$
- $\Rightarrow$ Copies required:

$$N \geq \left\lceil \frac{\pi}{\theta(V^\dagger U)} \right\rceil$$

- This guarantees $0 \in W((V^{\otimes N})^\dagger U^{\otimes N}) \Rightarrow$ diamond norm = 2 $\Rightarrow p_{\text{succ}} = 1$

Multi-shot Discrimination of Unitary Channels

- Assume for simplicity $\theta(V^\dagger U) = \pi/N$, where $N = w \cdot d$.
- In our setup N copies of the black box Φ can be composed in the **rectangle hybrid scheme**,
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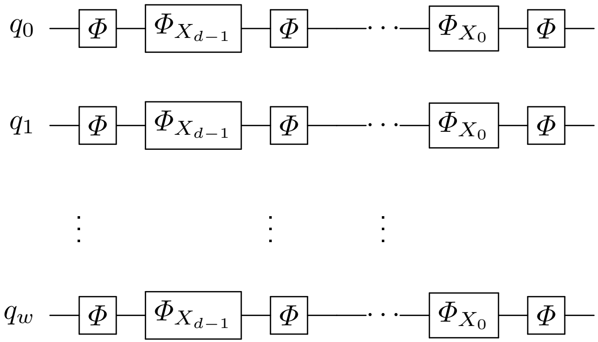

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- where $X_1, \ldots, X_{d-1}$ are arbitrary unitary matrices for mid-circuit processing.
- For some U and V processing is not necessary (but optimal always exists). Then

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- Strategy types:
  - $w = N, d = 1$: **Parallel**
  - $w = 1, d = N$: **Sequential**
  - General w, d: **Hybrid rectangular**

Discrimination of Unitary Channels on IBM Q

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  1. **Discriminator** – prepares state
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes w} + \alpha\,|1\rangle^{\otimes w})$$
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- Two discriminator and measurement implementations:
  - Standard CNOT-based GHZ construction (simple, but high transpilation overhead)
  - Optimized ECR-based version for Eagle R3 (lower gate count, better fidelity)

Example 1.

- We will distinguish between identity $\Phi_{\mathbf{1}}$ and $\Phi_{\mathsf{RZ}(\phi)}$ for

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- Measurement is then $E_0 = |\psi_0\rangle\langle\psi_0|$ and $E_1 = \mathbb{1} - E_0$, where $|\psi_0\rangle = |\psi\rangle$ after applying $\Phi_{\mathbf{1}}$

Discriminator Implementations: CNOT vs. ECR

Goal: Prepare a maximally entangled GHZ-like state on w qubits:

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We implemented two types of discriminator circuits to generate this:

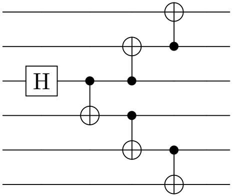

Figure: 6 qubit CNOT-based GHZ discriminator

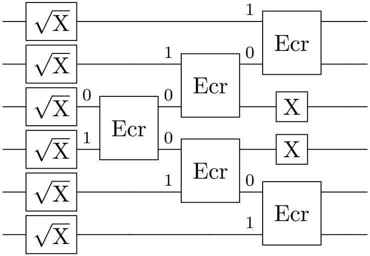

Figure: 6 qubit ECR-based discriminator optimized for Eagle R3

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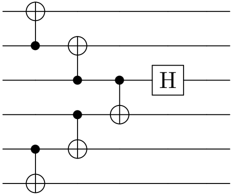
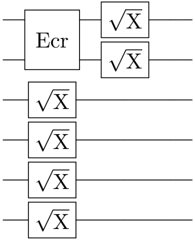


Figure: CNOT-based short measurement



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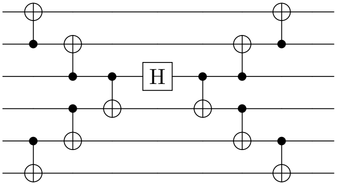
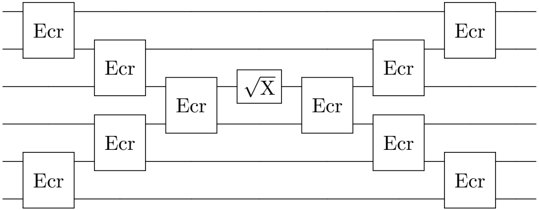


Figure: CNOT-based XOR measurement



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- **Key Insights:**
  - XOR measurement slightly outperforms short measurement when using ECR + Transpiler strategy.
  - Manual fixed mapping offers marginal benefits in this small system size.

Transpilation Strategy \ Measurement	Short	XOR
CNOT + Transpiler	88.8%	86.4%
ECR + Transpiler	83.8%	90.0%
ECR + Transpiler + Fixed Map	84.4%	85.3%
ECR + Fixed Map (No Opt.)	83.3%	85.6%

Table: Accuracy of different transpilation strategies for discrimination scheme on 6-qubits obtained on IBM Brisbane, using short and XOR measurement schemes. Each circuit was executed with 10,000 shots. Ambiguous measurement outcomes were randomly assigned.

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- **Key Insights:**
  - **Short measurement**: Best result from the default ECR + Transpiler.
  - **XOR measurement** greatly benefits from topology-aware design and fixed-qubit mapping, achieving up to **71.8% accuracy**.

Transpilation Strategy \ Measurement	Short	XOR
CNOT + Transpiler	43.3%	48.5%
ECR + Transpiler	55.0%	54.5%
ECR (Topol.) + Transpiler	36.1%	47.2%
ECR (Topol.) + Transpiler + Fixed Map	32.0%	71.5%
ECR (Topol.) + Fixed Map (No Opt.)	33.4%	71.8%

Table: Accuracy of different transpilation strategies for discrimination scheme on 11-qubit obtained on IBM Brisbane using short and XOR measurement schemes. Each circuit was executed with 10,000 shots. Ambiguous measurement outcomes were randomly assigned.

Impact of Circuit Structure: Sequential vs Parallel

- Comparison between **purely sequential** and **purely parallel** discrimination protocols.
- **Goal:** Distinguish between the identity and $R_z(\pi/N)$ gates using Short or XOR measurement schemes.

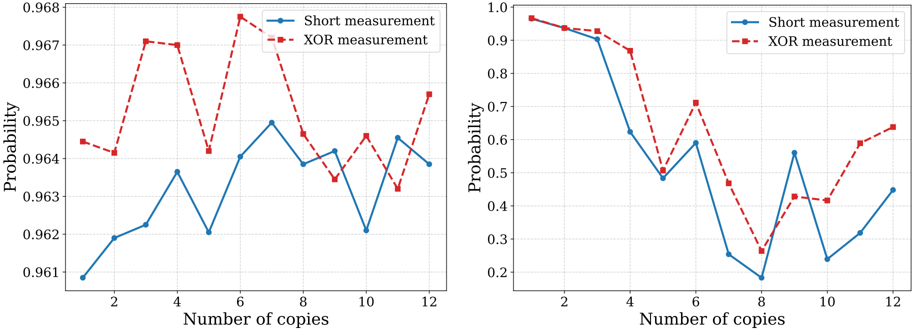
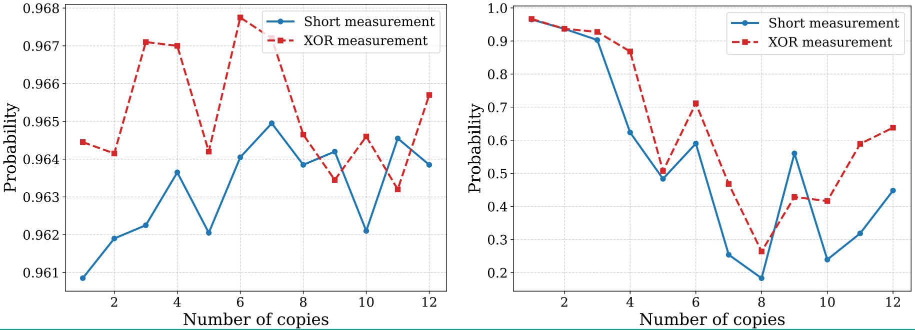


Figure: **(Left)** Sequential scheme. **(Right)** Parallel scheme. Solid blue: Short; dashed red: XOR.

Impact of Circuit Structure: Sequential vs Parallel

- Comparison between **purely sequential** and **purely parallel** discrimination protocols.
- **Goal:** Distinguish between the identity and $R_z(\pi/N)$ gates using Short or XOR measurement schemes.
- **Key Observation:**
  - Parallel circuits suffer more from noise due to entangling gates.
  - Sequential circuits, though deeper, maintain higher accuracy on real hardware.



Impact of Entanglement: Hybrid Schemes

- **Hybrid schemes** blend sequential and parallel discrimination by fixing total unitary applications N while varying the width of the scheme (qubits).

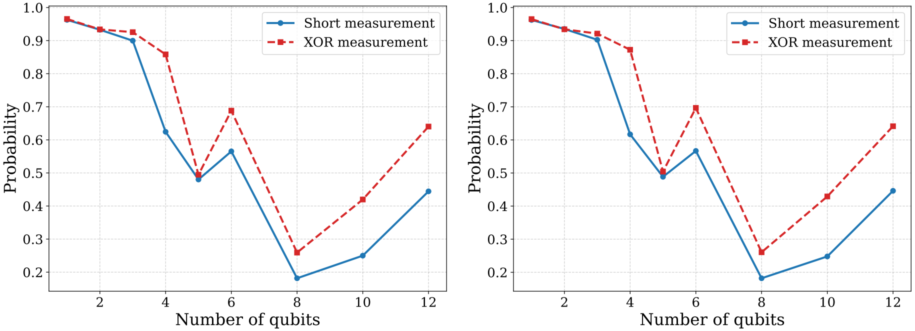
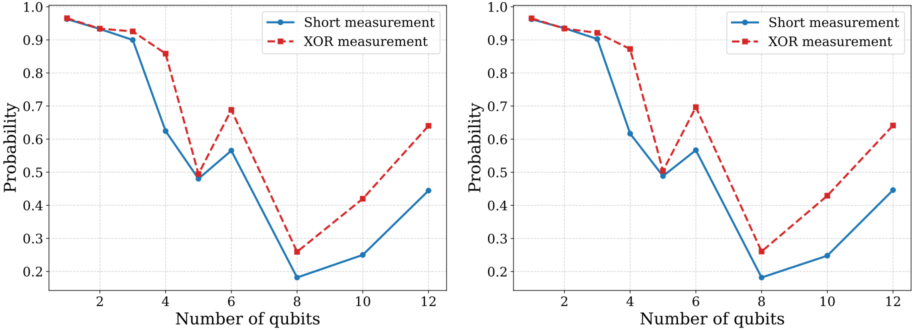


Figure: **(Left)** Hybrid scheme for $N = 240$. **(Right)** Hybrid scheme for $N = 1200$. Solid blue: short; dashed red: XOR.

Impact of Entanglement: Hybrid Schemes

- **Hybrid schemes** blend sequential and parallel discrimination by fixing total unitary applications N while varying the width of the scheme (qubits).
- **Key Observation:**
  - Increased entanglement (circuit width) leads to significantly higher error rates.
  - Confirms that **multi-qubit gate noise** is the dominant error source.



Post-Processing Correction of Bit-Flip Errors

- **Anomalous behavior:** On IBM Brisbane, circuits with specific number of qubits exhibit correlated bit-flip measurement errors across all qubits.
- **Correction:** Post-processing by swapping expected labels when the success probability drops below 0.5 restores accuracy.

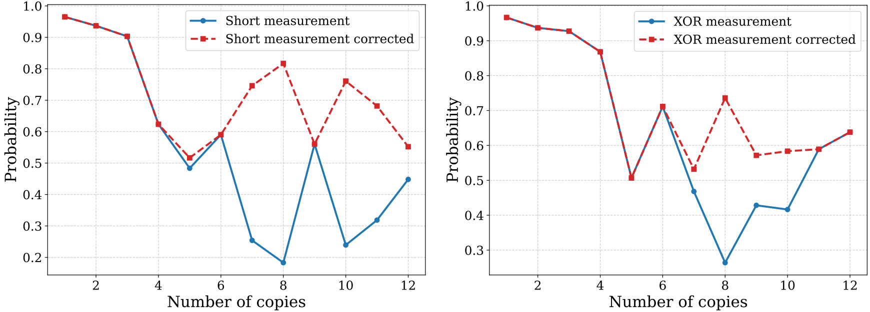
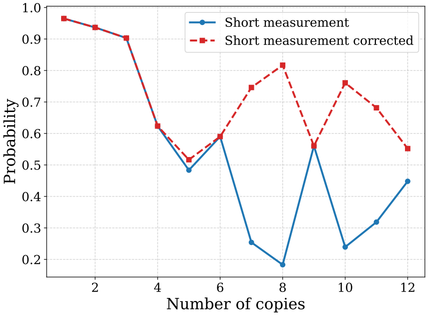
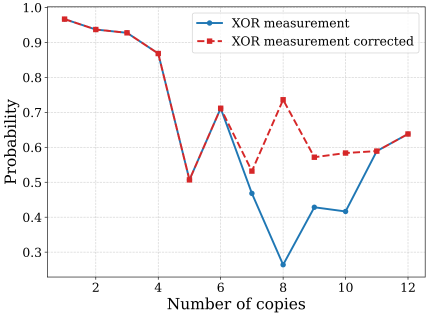


Figure: **(Left)** Corrected Short. **(Right)** Corrected XOR. Blue: original, Red: corrected.

Post-Processing Correction of Bit-Flip Errors

- **Anomalous behavior:** On IBM Brisbane, circuits with specific number of qubits exhibit correlated bit-flip measurement errors across all qubits.
- **Correction:** Post-processing by swapping expected labels when the success probability drops below 0.5 restores accuracy.
- **Verification:** Identical transpiled circuits executed on simulator produce perfect discrimination and **no such error is observed.**





Example 2.

- We will distinguish between Φ_U for $U = \sqrt{X}\,\mathrm{RZ}(\frac{-\pi}{2N})\sqrt{X}$ and Φ_V for $V = \sqrt{X}\,\mathrm{RZ}(\frac{\pi}{2N})\sqrt{X}$, where
$$\sqrt{X} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$
- **Observation:** $\sqrt{X}\,X\sqrt{X} = \mathbb{1}$, hence $U\,X\,U\,X = \sqrt{X}\,RZ(-\frac{\pi}{N})\,\sqrt{X}$.

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- **Observation:** $\sqrt{X}\,X\sqrt{X} = \mathbb{1}$, hence $U\,X\,U\,X = \sqrt{X}\,RZ(-\frac{\pi}{N})\,\sqrt{X}$.
- We use hardware-friendly mid-processing $X_i = X^{\otimes w}$, and for convince also pre-processing unitary operation $X_0 = (X\sqrt{X})^{\otimes w}$ and post-processing operation $X_d = (\sqrt{X}X)^{\otimes w}$.
- In this way, Example 2. is equivalent to Example 1.

Performance of hybrid schemes for Example 2.

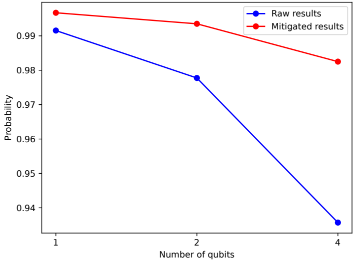

Figure: (a) Hybrid scheme for $N = 4$.

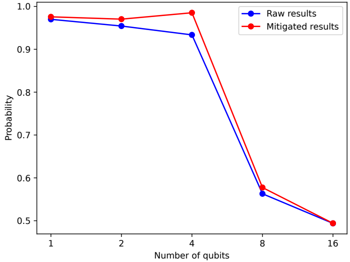

Figure: (b) Hybrid scheme for $N = 16$.

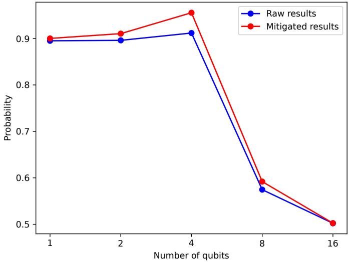

Figure: (c) Hybrid scheme for $N = 32$.

Probability of successful discrimination by hybrid rectangular scheme using the short measurement on IBM Q processor Brisbane. The blue line corresponds to no mitigation. The red line is after error mitigation using MThree package.

Performance of hybrid schemes for Example 2.

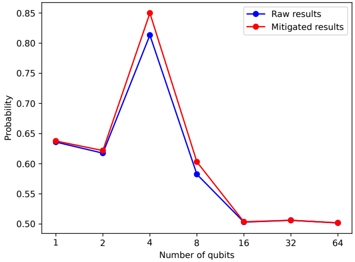

Figure: (a) Hybrid scheme for $N = 64$.

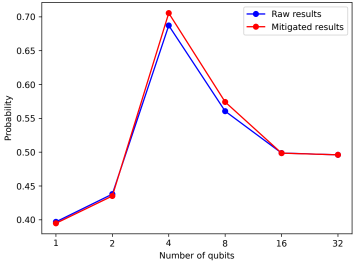

Figure: (b) Hybrid scheme for $N = 96$.

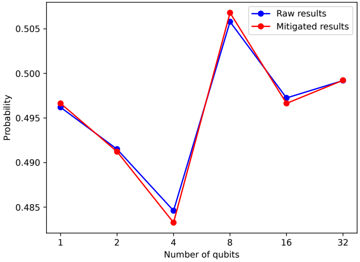

Figure: (c) Hybrid scheme for $N = 1024$.

Probability of successful discrimination by hybrid rectangular scheme using the short measurement on IBM Q processor Brisbane. The blue line corresponds to no mitigation. The red line is after error mitigation using MThree package.

Conclusion

- We have studied the discrimination of two quantum unitary channels and benchmarked various schemes for perfect discrimination between them.
- **Transpilation:**
  - Manual mapping helped on the 11-qubit layout with XOR-based measurement.
  - The overall benefit is often outweighed by the complexity and time consumption of manual mapping.
- **Practical Insight:**
  - Circuit geometries beyond square layouts may offer a more accurate reflection of the capabilities of the device
  - Purely parallel schemes typically perform poorly.
  - Purely sequential schemes work well only for a small number of copies.
- **Platform Anomaly:**
  - Systematic bit-flip errors with 5+ qubits in Example 1.
  - Probably hardware/software issue, not circuit design.

Acknowledgement

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Thank you for your attention

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